Statistics

Spring 2023

Lecture 9

Feb 19-8:47 AM

Consider the Following Sample

1, 2, 2, 2, 4

$n = 5$

Mode $= 2$

Range $= 4 - 1 = 3$

Midrange $= \frac{4+1}{2} = 2.5$

$\sum x = 1 + 2 + 2 + 2 + 4 = \boxed{11}$

$\bar{x} = \frac{\sum x}{n} = \frac{11}{5} = \boxed{2.2}$

$\sum x^2 = 1^2 + 2^2 + 2^2 + 2^2 + 4^2 = \boxed{29}$

$$S^2 = \frac{n\sum x^2 - (\sum x)^2}{n(n-1)} = \frac{5 \cdot 29 - 11^2}{5(5-1)} = \frac{24}{20} = \boxed{1.2}$$

$$S = \sqrt{S^2} = \sqrt{1.2} \approx \boxed{1.095}$$

Estimate S $\quad S \approx \frac{\text{Range}}{4} = \frac{3}{4} = \boxed{.75}$

Feb 21-7:16 AM

Now clear all lists

Store 1, 2, 2, 2, 40 in L1

Use 1-Var stats with L1 to find

$\bar{x} = 9.4$

$S \approx 17.111$

$n = 5$

find S^2 in reduced fraction

$\boxed{\text{VARS}}$ $\boxed{\text{5: Statistics}}$ $\boxed{3: S_x}$ $\boxed{x^2}$

$\boxed{\text{Math}}$ $\boxed{\text{1: ▶Frac}}$ $\boxed{\text{Enter}}$

$S^2 = \frac{1464}{5}$

when data elements are close to $\bar{x}$ $\Leftrightarrow$ S is small.

when data elements are more spread out from $\bar{x}$ $\Leftrightarrow$ S is big.

when all data elements are the same $\Leftrightarrow$ $S = 0$ and are equal to $\bar{x}$.

Sample standard deviation $S \geq 0$

$S = \sqrt{S^2}$ ← Sample Variance

Feb 21-7:24 AM

Suppose $n = 6$, $\sum x = 48$, $\sum x^2 = 384$

1) $\bar{x} = \frac{\sum x}{n} = \frac{48}{6} = \boxed{8}$

2) $S^2 = \frac{n\sum x^2 - (\sum x)^2}{n(n-1)} = \frac{6 \cdot 384 - 48^2}{6(6-1)} = \frac{0}{30} = \boxed{0}$

3) $S = \sqrt{S^2} = \sqrt{0} = \boxed{0}$

4) Draw conclusion about data elements.

Since $S = 0$, all data elements are the same and equal to $\bar{x} = 8$

Feb 21-7:34 AM

I randomly selected 12 students. Here are their ages:

32	30	18	25
20	19	28	40
35	24	26	27

1) $n = 12$

2) Range $= 40 - 18 = 22$

3) Mode No mode

4) Estimate $S \approx \frac{\text{Range}}{4} = \frac{22}{4} = \boxed{5.5}$

Store this sample in L1, use 1-Var Stats with L1 only to find

5) $\Sigma x = 324$

6) $\Sigma x^2 = 9224$

7) $\bar{x} = 27$

8) S round to 3-decimal places $= 6.578$

9) S^2 in reduced fraction

VARS

5: | 3:

x^2 | Math

1: | Enter

$\boxed{S^2 = \frac{476}{11}}$

$\left.\begin{array}{l} \text{Min} = 18 \\ Q_1 = 22 \\ \text{Med} = 26.5 \\ Q_3 = 31 \\ \text{Max} = 40 \end{array}\right\}$ 5-Number Summary

Feb 21-7:38 AM

Min = 18

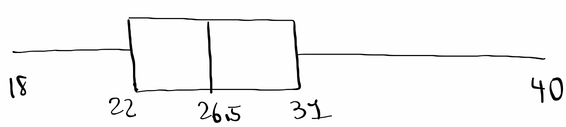
$Q_1 = 22$

Med = 26.5

$Q_3 = 31$

Max = 40

Draw Box Plot

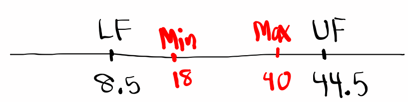


Find IQR $= Q_3 - Q_1 = 31 - 22 = \boxed{9}$

Upper Fence $= Q_3 + 1.5(IQR) = 31 + 1.5(9) = \boxed{44.5}$

Lower Fence $= Q_1 - 1.5(IQR) = 22 - 1.5(9) = \boxed{8.5}$

Discuss outliers



Nothing above upper fence

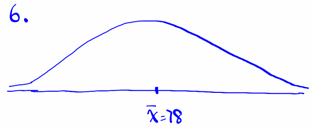
&

Nothing below lower fence

$\Rightarrow$ No outliers

Feb 21-7:50 AM

I randomly selected 60 exams. Scores were symmetric with a bell-shape dist with $\bar{x} = 78$ and $S = 6$.



Use Empirical Rule to find

1) 68% Range $\Rightarrow \bar{x} \pm S = 78 \pm 6 \Rightarrow \boxed{72 \text{ to } 84}$

2) 95% Range $\Rightarrow \bar{x} \pm 2S = 78 \pm 2(6) \Rightarrow \boxed{66 \text{ to } 90}$

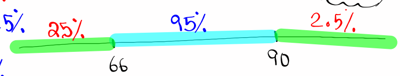
Usual Range

3) 99.7% Range $\Rightarrow \bar{x} \pm 3S = 78 \pm 3(6) \Rightarrow \boxed{60 \text{ to } 96}$

4) what % of Scores were above 90? 2.5%

100% − 95% = 5%

5% ÷ 2 = 2.5%



5) How many of Scores were above 90?

what is 2.5% of 60?

$x = 2.5(.01) \cdot 60$

$\boxed{x = 1.5}$ about 2 Scores were above 90.

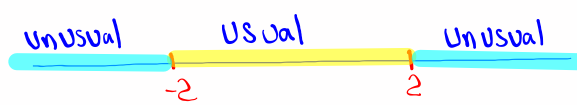
Feb 21-7:57 AM

Z Scores

Always round to 3-decimal places

When $-2 \le Z \le 2 \Rightarrow$ Data element is usual.

When $Z < -2$ or $Z > 2 \Rightarrow$ Data element is Unusual.



$$Z = \frac{x - \bar{x}}{S}$$

It is a value that indicates how many standard deviation is the data element above or below the mean.

It allows us to standardize data elements and compare data elements from different samples.

Feb 21-8:07 AM

Suppose $\bar{x} = 78$ and $S = 5$.

1) Find Z-Score for $x = 90$.

$$Z = \frac{x - \bar{x}}{S} = \frac{90 - 78}{5} = \boxed{2.4}$$

Since $Z > 2$, 90 is unusual.

2) Find x when $Z = -1.4$.

$$Z = \frac{x - \bar{x}}{S} \qquad -1.4 = \frac{x - 78}{5}$$

Cross-multiply

$$x - 78 = 5(-1.4)$$

$$x = 78 - 7$$

Since $-2 \le Z \le 2$ → $\boxed{x = 71}$ is a usual element.

Feb 21-8:12 AM

Marc got 92 on exam 1, and 80 on exam 2.

To compare these two Scores, we need to compare Z-Scores, So we need $\bar{x}$ & S for each exam.

Exam 1:
$\bar{x} = 84$, $S = 10$

$$Z = \frac{x - \bar{x}}{S} = \frac{92 - 84}{10} = \boxed{.8} \text{ usual}$$

Exam 2:
$\bar{x} = 75$, $S = 2$

$$Z = \frac{x - \bar{x}}{S} = \frac{80 - 75}{2} = \boxed{2.5} \text{ unusual}$$

He did better in exam 2.

Feb 21-8:19 AM

Clear all lists

Store the following in L1

68	75	70	55	80
90	73	70	95	82
100	95	58	60	69
74	88	98	90	80

Now Sort L1 from smallest to largest

$\boxed{\text{STAT}}$ Edit

$\boxed{\text{2:SortA(}}$ L1 $\boxed{\text{Enter}}$

Now View L1 & make STEM Plot

$\boxed{\text{2nd}}$ $\boxed{\text{1}}$ $\boxed{\text{Enter}}$

5	5 8
6	0 8 9
7	0 0 3 4 5
8	0 0 2 8
9	0 0 5 5 8
10	0

Feb 21-8:26 AM